<Exploring Fuzzy Logic to Combine Foot Type and Pointe Shoes>

(Tara Webb)

Executive Summary

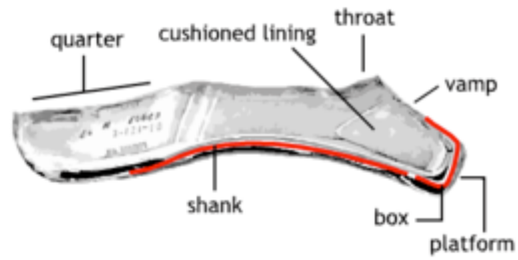

Figure 1 (“Pointe Shoes,” n.d.)

Pointe shoes, also known (less correctly) as toe shoes, are a special type of shoe used by ballet dancers for pointework. Dancers now use satin pointe shoes with a hard but pliable shank and a box made up of layers of canvas, hessian, paper and glue. There are many different types of pointe shoes, and each fits the dancer in a different way.

The shoes have two important structural features that allow the dancer to dance on the tips of her toes:

> - the *box* is a section of burlap stiffened with glue, that encases and supports the dancer's toes. The end of the box is covered with satin and flattened into a *platform*, upon which the dancer can balance.
>
> - the *shank* is a strengthened piece of material (usually many layers of glue-hardened burlap, thick leather, or sometimes plastic) running near to the length of the dancer's sole. It provides support to the arch of her foot as she stands en pointe. A hard shank is good for dancers who have an extremely high arch where a softer shank is good for dancers who have a flatter foot (“Pointe Shoes,” n.d.).

Another important feature of the pointe shoe is the vamp length. A short vamp is good for dancers who have short toes and a longer vamp is good for dancers who have longer toes.

Pointe shoes are an important component to being a successful dancer. The barrage of pointe shoes in the dance catalogs can make it extremely confusing to even begin to figure out which shoe is best for you. I thought it would be interesting and beneficial to find a way to put the shoes into groups so there would be no mystery. A prospective pointe shoe buyer would have a variety of options nicely grouped into a self-explanatory manner.

Figure 2 (“What is My Foot Type,” n.d.)

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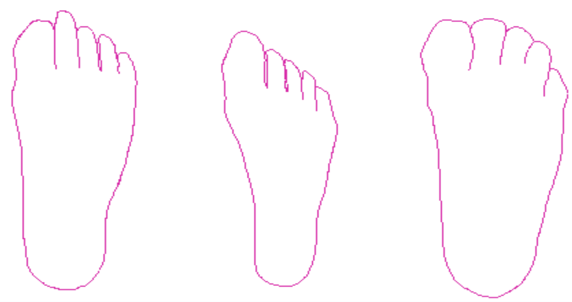

Greek/Morton's Foot

This foot type has a second toe that is longer than all the others. The width tends to be narrow to medium.

Egyptian Foot

This foot type has a long first toe and the rest of the toes taper. The width tends to be narrow to medium.

Giselle/Peasant Foot

This foot type has at least three toes the same length (sometimes more) and the toes tend to be short. It tends to be well-suited for pointework. The width tends to be medium to wide.

Figure 3 (“Feet and Foot Types,” n.d.)

Grouping these shoes will involve matching all the shoes in an optimal way to the different foot types. I have decided to combine the two different ways of measuring foot type to give nine different kinds of feet (see Fig. 2 and 3):

- Greek, Flat
- Egyptian, Flat
- Giselle, Flat
- Greek, Normal
- Egyptian, Normal
- Giselle, Normal
- Greek, High-Arch
- Egyptian, High-Arch
- Giselle, High-Arch

The attributes I will focus on in grouping will be the shank, the box, and the vamp, as these are most crucial to a proper fitting. I will be using the major brands of pointe shoes, including Freed, Capezio, Grishko, and Bloch. In each of these brands, there are several different styles of pointe shoes which would be used for my analysis.

To make these groupings, I researched fuzzy logic. The beginnings of fuzzy logic were first given in 1965 by Lotfi Zadeh. Humans use fuzzy logic in their everyday life. Human beings have the ability to take in and evaluate all sorts of information from the physical world they are in contact with and to mentally analyze, average and summarize all this input data into an optimum course of action. This is what fuzzy logic does.

Humans evaluate input from their surroundings in a fuzzy manner, whereas machines/computers obtain precise appearing values, such as 112 degrees F, obtained with a transducer and an analog to digital converter. The computer input would be the computer measuring, let's say, 112 degrees F. The human input would be a fuzzy feeling of being too warm.

The human says, "The shower water is too hot." The computer as a result of analog input measurement says, "The shower water is 112 degrees F and 'If-Then' statements in my program tell me the water is too warm." A human says, "I see two tall people and one short one." The computer says, "I measure two people, 6' 6" and 6' 9", respectively, and one person 5' 1" tall, and 'If-Then' statements in my program tell me there are two tall people and one short person (Sowell, n.d.)."

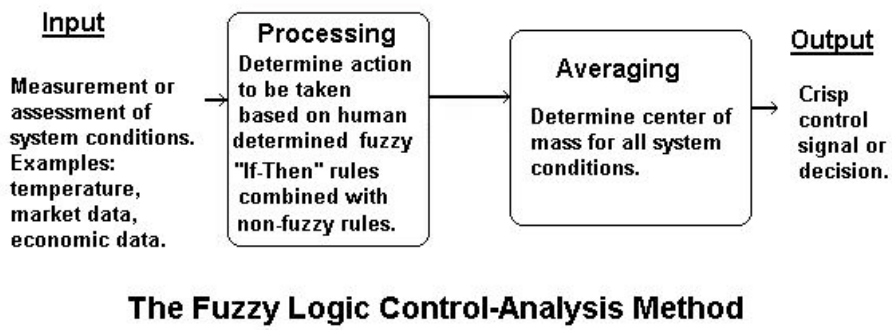

Figure 4 (Sowell, n.d.)

A key part of fuzzy logic is the if-then statement. An example would be: “If the cart is on the right, then pull.”

Mathematically, the data is turned into a percentage of how much it belongs to the “if” part of the statement. Then the action that is to be taken (the “then” part) is also turned into a percentage of belonging. For example, the data could have a percentage of belonging of 85%. These two percentages are then combined to give an output. (see Fig. 4)

A typical example used to explain this is a cart on a track. The cart’s position can be described in terms of fuzzy sets as left, middle, or right. The action taken on the cart to get it to be in the middle could be described in terms of fuzzy sets as pull, none, and push. One could then get the percentages of belonging from these fuzzy sets.

Then, some rules could be constructed. A few examples for this could be:
1) If left, then push.
2) If right, then pull.
3) If middle, then none.

The next step would be to use these rules to combine the percentages. This would then lead to a final decision on which action to take (Michalewicz & Fogel, 2002).

To use this method, a computer program would have to be written using the components shown in Fig. 4.

To utilize this method for my problem, I would be using the different foot types and kinds of pointe shoes for the if-then statements in the program. The data for the pointe shoes would come from

Discount Dance Supply's Catalog (www.discountdance.com). From this catalog, I would use the shank strength (soft, medium, or hard), the vamp length (short or long), and the shape of the box (tapered or broad).

I believe that this approach would be the optimal way to solve my problem of matching different foot types to different pointe shoes. In research, I could not find another example of this kind of analysis. However, since every person's feet are different, this way of having a percentage of belonging would be the most useful way to approach this problem. Therefore, I recommend that this method be used to solve this problem optimally.

Problem Description

Pointe shoes are an important component in being a successful dancer. The barrage of pointe shoes in the dance catalogs can make it extremely confusing to even begin to figure out which shoe is best for you. I thought it would be interesting and beneficial to find a way to put the shoes into groups so there would be no mystery. A prospective pointe shoe buyer would have a variety of options nicely grouped into a self-explanatory manner.

To do this, I have researched the use of fuzzy logic and its appropriateness to this problem.

Analysis Technique

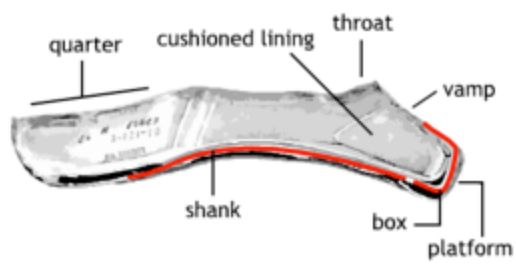

Figure 1 ("Pointe Shoes," n.d.)

Pointe shoes, also known (less correctly) as toe shoes, are a special type of shoe used by ballet dancers for pointework. They developed from the desire to appear weightless onstage and have evolved to allow extended periods of movement on the tips of the toes (*en pointe*). Pointe shoes are normally worn only by female dancers, though male dancers may wear them for certain roles, such as the ugly stepsisters in *Cinderella*, Bottom in *A Midsummer Night's Dream*, or men in dance companies such as Les Ballets Trockadero de Monte Carlo.

As dance extended into the 1800s, the emphasis on technical skill increased, as did the desire to dance en pointe without the aid of wires. When Marie Taglioni first danced *La Sylphide* en pointe, her shoes were nothing more than satin slippers, darned at the ends. The sole was made of leather and the sides and toe were darned to keep its shape. Dancers relied heavily on their own strength, in the feet and ankles, without the support of a hard pointe shoe. They most likely padded the toes for some comfort.

Dancers now use satin pointe shoes with a hard but pliable shank and a box made up of layers of canvas, hessian, paper and glue. There are many different types of pointe shoes, and each fits the dancer in a different way. The pointe shoe should be tight, with only a pinch of cloth at the heel when the pointe shoe is *en pointe*. Two ribbons wrap around the dancer's ankle, sometimes with an elastic band that wraps around from the back of the heel, to the front, and then back to the back of the heel; or across the instep as with ballet shoes; or is attached with a loop on the heel which the ribbons pass through. The shank of the shoe comes in two different sizes, 3/4 and full shank. There is a wide variety of pointe shoes that have different attributes and longevity.

The shoes have two important structural features that allow the dancer to dance on the tips of her toes:

> - the *box* is a section of burlap stiffened with glue, that encases and supports the dancer's toes. The end of the box is covered with satin and flattened into a *platform*, upon which the dancer can balance.
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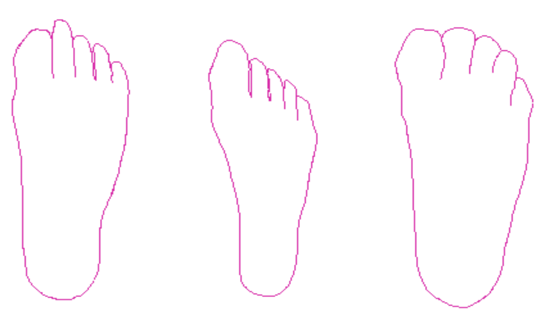
Another important feature of the pointe shoe is the vamp length. A short vamp is good for dancers who have short toes and a longer vamp is good for dancers who have longer toes.

In determining the foot type of the dancer and fitting for pointe shoes, there are two different approaches to consider. The first is what is known as "the wet test." In this, one puts their foot into water or paint to see what kind of arch they have.

Flat Feet Normal Feet High Arched Feet

Figure 2 ("What is My Foot Type," n.d.)

The second consideration is the shape of the toes and width of the foot.



Greek/Morton's Foot	Egyptian Foot	Giselle/Peasant Foot
This foot type has a second toe that is longer than all the others. The width tends to be narrow to medium.	This foot type has a long first toe and the rest of the toes taper. The width tends to be narrow to medium.	This foot type has at least three toes the same length (sometimes more) and the toes tend to be short. It tends to be well-suited for pointework. The width tends to be medium to wide.

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Computer based fuzzy logic machine control is like human fuzzy logic control, but there is a difference when the nature of the computer's input is considered.

Humans evaluate input from their surroundings in a fuzzy manner, whereas machines/computers obtain precise appearing values, such as 112 degrees F, obtained with a transducer and an analog to digital converter. The computer input would be the computer measuring, let's say, 112 degrees F. The human input would be a fuzzy feeling of being too warm.

The human says, "The shower water is too hot." The computer as a result of analog input measurement says, "The shower water is 112 degrees F and 'If-Then' statements in my program tell me the water is too warm." A human says, "I see two tall people and one short one." The computer says, "I measure two people, 6' 6" and 6' 9", respectively, and one person 5' 1" tall, and 'If-Then' statements in my program tell me there are two tall people and one short person."

Even though transducer derived, measured inputs for computers appear to be more precise, from the point of input forward we still use them in a fuzzy logic method approach that follows our fuzzy, human approach to control.

For a human, if the shower water gets too warm, the valve handle is turned to make the temperature go down a little. For a computer, an "If-Then" statement in the program would initiate the lowering of temperature based on a human provided "If-Then" rule, with a command output operating a valve (Sowell, n.d.).

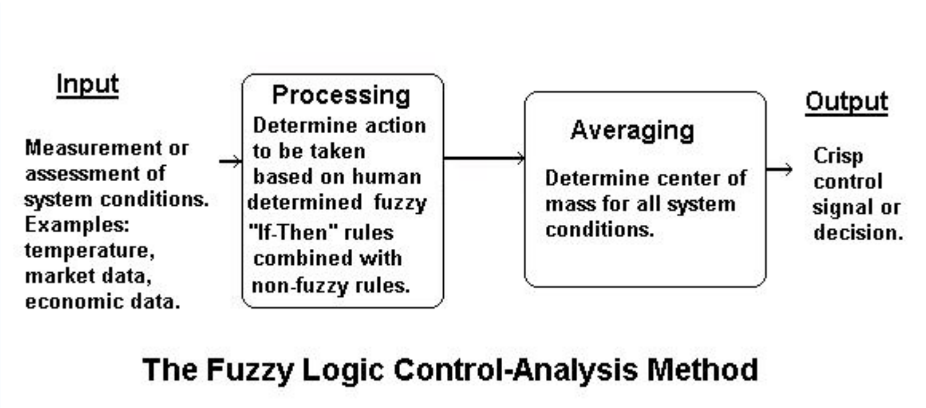

Figure 4 (Sowell, n.d.)

To create a personal computer based fuzzy logic control system:

1. Determine the inputs.

2. Describe the cause and effect action of the system with "fuzzy rules" stated in plain English words.

3. Write a computer program to act on the inputs and determine the output, considering each input separately. The rules become "If-Then" statements in the program.

4. In the program, use a weighted average to merge the various actions called for by the individual inputs into one crisp output acting on the controlled system. (In the event there is only one output, then merging is not necessary, only scaling the output as needed.) (Sowell, n.d.)

Mathematically, the foundations of fuzzy logic can be thought of as an extension of set theory. In a classic set, the elements are either in the set or not in the set. A set can be described as a membership function, $m_A(x)$, defined over some “universe of discourse.” The function would take on the value 1 when x is an element of the set and 0 when x is not an element. With a fuzzy set, $m_A(x)$ could take on values other than 0 or 1. It could take on value in the range of [0, α], where the value assumed indicates the degree of membership of x in the set. The range of values is often scaled to be all real numbers between 0 and 1, inclusive. A membership of 0 means that the element absolutely does not belong to the set and a membership of 1 means that the element definitely does belong to the set. Any values between these indicate how much the element belongs to the set.

An example of a membership function describing the fuzzy set of ages that are considered old might be:

$m_A(x) = 0$ if $x \leq 50$
$m_A(x) = (1 + (25/(x-50)^2)^{-1}$ if $x \geq 50$

The choice of a membership function is subjective. Each person can choose one that captures their own perspective. This means that fuzzy systems can reflect personal biases and can be made to fit an individual's viewpoint.

The extension of this is how fuzzy sets are related, which leads to the if-then statement: if A then B. This A and B imply fuzzy sets and the if-then implies a relationship between these sets. Because the "universe of discourse" does not need to be the same for A and B, the new "universe of discourse" that describes this relationship will be the Cartesian product A X B of all pairs (*a,b*) where *a* belongs to A and *b* belongs to B. If R is a fuzzy relation from A to B, R is a fuzzy subset of A X B where every pair (*a,b*) has some associated membership value $m_R(a,b)$. The membership values for elements in A are given in ordered pairs (a, $m_A(a)$) and similarly the values for elements in B are given as (b, $m_B(b)$). Then, R = A X B is defined to take on a membership that equals the min{ $m_A(a)$, $m_B(b)$ } for each (*a,b*) pair. This R can be described by a matrix.

After all of these if-then statements are constructed and the membership functions are calculated, then the next step is to aggregate all of the results from each of these statements. This is done by taking the maximum membership for each state in the "then" part of the if-then statement across all the results of each rule. This gives a new fuzzy set.

The last part to consider is to defuzzify this fuzzy set to get a single crisp output value. The most common way to do so is to use the "center of area" method, which uses a ratio to arrive at the final value.

A typical example used to explain this is a cart on a track. The cart's position can be described in terms of fuzzy sets as left, middle, or right. The action taken on the cart to get it to be in the middle could be described in terms of fuzzy sets as pull, none, and push. One could then get membership functions from these fuzzy sets.

Then, some rules could be constructed. A few examples for this could be:

4) If left, then push.
5) If right, then pull.
6) If middle, then none.

The next step would be to use these rules to combine the membership functions. This would then lead to defuzzification and a final decision on which action to take (Michalewicz & Fogel, 2002).

To utilize this method for my problem, I would be using the different foot types and kinds of pointe shoes for the if-then statements in the program. The data for the pointe shoes would come from Discount Dance Supply's Catalog (www.discountdance.com). From this catalog, I would use the shank strength (soft, medium, or hard), the vamp length (short or long), and the shape of the box (tapered or broad).

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Assumptions

- Foot types are very different and there are many combinations.
- In using fuzzy logic, there will not be precise groupings found, but there will be groupings which are optimal in most cases.

Results

To utilize fuzzy logic for my problem, I would be using the different foot types and kinds of pointe shoes for the if-then statements in the program. The data for the pointe shoes would come from Discount Dance Supply's Catalog (www.discountdance.com). From this catalog, I would use the shank strength (soft, medium, or hard), the vamp length (short or long), and the shape of the box (tapered or broad).

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Issues

In my research, I could not find another example of fuzzy logic being used for exactly this problem. However, I am confident in the research I have done that this method is the optimal way to solve my problem.

Also, I would have liked to actually have found an algorithm to actually solve my problem. Because of my lack of experience with programming, I was not able to do this.

Appendices

Feet and Foot Types. Retrieved October 8, 2006, from www.dancer.com/**foot**type

Fuzzy Logic. Retrieved October 8, 2006 from http://en.wikipedia.org/wiki/Fuzzy_logic

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